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SS433 and Hydrogen Spectrum Beyond the Paschen–Back Region

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We examine the hydrogen spectrum in intense magnetic fields beyond the Paschen–Back region. Unlike in the Zeeman and Paschen–Back regions, the perturbations from the normal hydrogen spectrum increase as the magnetic field *decreases*!

1. INTRODUCTION

SS433 is thought to contain a neutron star and a companion (13-day binary period (Abell and Margon, 1979). A simultaneously red- and blue-shifted hydrogen spectrum is an unusual feature of this system. Furthermore, the blue-shifted spectrum periodically becomes red-shifted and vice versa every 164 days. The narrow spectral lines suggest a well-collimated emission region, while the magnitudes of the red and blue shifts indicate source region speeds of $0.26c$ (Abell and Margon, 1979).

Such behavior can be accounted for by radiating material moving away from the poles of a rotating neutron star (Millgram, 1981; Margon, 1983; Fabian and Rees, 1979) or by infalling material (Cohen and Struble, 1980, 1982; Cohen *et al.*, 1982). With the infalling model, the radiating region is collimated by the converging magnetic field lines near the neutron star magnetic poles; and is accelerated by the neutron star gravitational field.

The alternating red- and blue-shifted spectra can be accounted for by both models. However, the infall model has an acceleration and a collimation mechanism. In an attempt to find observational differences between the two models, we investigate possible differences between the hydrogen spectra.

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Previously, it was found that for very strong magnetic fields far beyond the Paschen–Back region, the hydrogen spectrum is essentially the same as that of the unperturbed case (Cohen *et al.*, 1988). The possibility that the hydrogen spectral levels may be increasingly perturbed as the fields become weaker is investigated here.

2. HYDROGEN SPECTRUM

In strong magnetic fields $B > 10^{10}$ G the Schrödinger equation gives transverse energy states (Landau levels) with wide spacing ($E = \hbar\omega$, where $\omega = eB/mc$ or $E_e \approx 1.9 \times 10^4$ eV $\times B/10^{12}$ G). These Landau levels are perturbed due to the Coulomb interaction. Also, in the lowest magnetic state of the system (Cohen *et al.*, 1988; Landau and Lifshitz, 1981), the Schrödinger equation for longitudinal motion of the electron (in the z direction, along the magnetic field lines) is given by

$$-\frac{\hbar^2}{2m}\frac{1}{z^2}\frac{d}{dz}\left(z^2\frac{d\phi}{dz}\right) + V(z)\phi = E\phi \tag{1}$$

where

$$V(z) \approx -e^2(z^2 + a^2)^{-1/2} \tag{2}$$

or

$$\begin{aligned} V(z) &\approx \frac{-e^2}{z}, \qquad z \geq a \\ V(z) &\approx \frac{-e^2}{a}, \qquad z < a \end{aligned} \tag{3}$$

with the cyclotron radius $a = (\hbar c/eB)^{1/2} = 0.0256667(10^{12}/B)^{1/2}$ Å.

Here, the approximate potential (3) results from the electron moving in magnetic orbital states and thus not approaching arbitrarily close to the nucleus.

It is of interest to examine the energy level shifts from those of normal hydrogen atom states (Landau and Lifshitz, 1981)

$$E_n = \frac{-13.6}{n^2} \text{ eV} \tag{4}$$

where $n = 1, 2, 3, \ldots$.

A first-order perturbation analysis gives

$$\Delta E_n = \langle n|\Delta V|n\rangle \tag{5}$$

with

$$\Delta V = 0; \qquad z \geq a$$
$$\Delta V = -e^2 \left(\frac{1}{a} - \frac{1}{z}\right) \tag{6}$$

where $e = 4.80 \times 10^{-10}$ esu.

Thus, corresponding to the energy levels (4) for $n = 1, 2, 3$ we have

$$\Delta E_{n,0} = e^2 \int_0^a R^*_{n,0} \left(\frac{1}{r} - \frac{1}{a}\right) R_{n,0} r^2 \, dr \tag{7}$$

with

$$R_{1,0} = \left(\frac{2}{a_0^{3/2}}\right) e^{\frac{-r}{a_0}}$$
$$R_{2,0} = \left(\frac{1}{2\sqrt{2}\, a_0^{3/2}}\right)\left(2 - \frac{r}{a_0}\right) e^{\frac{-r}{2a_0}} \tag{8}$$
$$R_{3,0} = \left(\frac{2}{81\sqrt{3}\, a_0^{3/2}}\right)\left(27 - 18\frac{r}{a_0} + \frac{2r^2}{a_0^2} e^{-r/3a_0}\right)$$

where $a_0 = 0.528$ Å is the Bohr radius.

The integrals (7) can be obtained by integrating $\int_0^a e^{ax}\, dx$ and differentiating with respect to a or by tables of integrals (e.g., Gradshtein and Ryzhik, 1965).

The results are as follows:

$$\frac{\Delta E_1}{|E_1|} = 2\left[e^{-2x}\left(1 + \frac{1}{x}\right) + \left(1 - \frac{1}{x}\right)\right] \tag{9a}$$

with $x = a/a_0$; and

$$\frac{\Delta E_2}{|E_2|} = \left[e^{-x}\left(x^2 + 2x + 6 + \frac{8}{x}\right) + 2 - \frac{8}{x}\right] \tag{9b}$$

$$\frac{\Delta E_3}{|E_3|} = \frac{8}{2187}\left[e^{-2x/3}(729x + 243x^2 - 27x^3 + 9x^4 + 2733.75) + (e^{-2x/3} - 1)\frac{4920.75}{x} + 546.75\right] \tag{9c}$$

Table I. Energy Level Shifts $\Delta E_n/|E_n|$ vs. Magnetic Field B

B (G)	a (Å)	$x=a/a_0$	$\Delta E_1/\|E_1\|$	$\Delta E_2/\|E_2\|$	$\Delta E_3/\|E_3\|$
10^{12}	0.0257	0.0467	3.0×10^{-3}	1.504×10^{-3}	1.002×10^{-3}
10^{11}	0.0813	0.154	0.0272	1.354×10^{-2}	9.04×10^{-3}
10^{10}	0.257	0.487	0.1988	0.0978	0.065

In the limit as B becomes large, a becomes small and the results take the limiting values $\Delta E_n/|E_n|=\frac{4}{3}x^2/n$ $(n=1, 2, 3, \ldots)$, i.e.,

$$\frac{\Delta E_1}{|E_1|}=\frac{4}{3}x^2 \qquad \frac{\Delta E_2}{|E_2|}=\frac{2}{3}x^2 \qquad \frac{\Delta E_3}{|E_3|}=\frac{4}{9}x^2 \tag{10}$$

The numerical values of the energy level shifts $\Delta E_n/|E_n|$ as a function of magnetic field B are given in Table I, where a (Å) $=0.0257\times(10^{12}/B)^{1/2}$; $x=0.0487\times(10^{12}/B)^{1/2}$; $\Delta E_n/|E_n|=(0.0032/n)\ (10^{12}/B)$.

These results are in accord with the work of other investigators (Glossman and Castro, 1988; Ivanov, 1988; Garstang, 1977; Haines and Roberts, 1969; Rösner *et al.*, 1984).

3. DISCUSSION

Here we have estimated the theoretical shift in the energy states in a strong magnetic field as compared to no magnetic field. As stated previously (Cohen *et al.*, 1988), in the limit of very strong magnetic fields the emission spectrum of hydrogen is identical to that of the zero-magnetic-field case. We have examined the spectra when the field is not arbitrarily large and have found a small perturbation from the usual hydrogen spectrum. It is interesting to note that *the perturbation grows as the field becomes weaker*, unlike in the Zeeman and Paschen–Back regions.

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